

An Effective Handover Analysis for the Randomly Distributed Heterogeneous Cellular Networks

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Abstract

Handover rate is one of the most important metrics to instruct mobility management and resource management in wireless cellular networks. In the literature, the mathematical expression of handover rate has been derived for homogeneous cellular network by both regular hexagon coverage model and stochastic geometry model, but there has not been any reliable result for heterogeneous cellular networks (HCNs). Recently, stochastic geometry modeling has been shown to model well the real deployment of HCNs and has been extensively used to analyze HCNs. In this paper, we give an effective handover analysis for HCNs by stochastic geometry modeling, derive the mathematical expression of handover rate by employing an infinitesimal method for a generalized multi-tier scenario, discuss the result by deriving some meaningful corollaries, and validate the analysis by computer simulation with multiple walking models. By our analysis, we find that in HCNs the handover rate is related to many factors like the base stations' densities and transmitting powers, user's velocity distribution, bias factor, pass loss factor and etc. Although our analysis focuses on the scenario of multi-tier HCNs, the analytical framework can be easily extended for more complex scenarios, and may shed some light for future study.

Keywords

Stochastic geometry modeling, handover rates, heterogeneous cellular network.

1 Introduction

With the dramatically increasing of wireless traffic as well as the population of wireless terminals, the traditional homogeneous cellular network cannot provide sufficient bandwidth for all the wireless terminals. In response to the capacity challenges, smaller coverage base stations (BSs) are deployed in hotspots to offload and have a range of tens of meters to several hundred meters. This brings heterogeneity to traditional cellular network and gives birth to the heterogeneous cellular network (HCN). Heterogeneity is expected to be a key feature of the next generation of cellular networks, and an essential means for providing higher network capacity as well as expanded indoor and cell-edge coverage. In general, HCNs comprise a conventional cellular network overlaid with a diverse set of lower-power BSs such as micro cells, femtocells and perhaps relay BSs.

The BSs in different tiers of HCNs (the tiers of BSs are ordered by transmit power) may share the same spectra and have different coverage. Handover happens when a user leaves the coverage of its serving BS and handover rate is defined as the number of handovers per unit time. The handover in HCNs can be divided into two types: horizontal handover and vertical handover. Horizontal handover is the handover between two BSs in the same tier and vertical handover is the handover between two BSs in different tiers. Compared with horizontal handovers, vertical handovers are more difficult to implement because the HCNs may be deployed by different service providers. Thus, to implement vertical handovers, extra communication overhead between the the HCNs is essential. Moreover, vertical handovers would lead to additional transfer delay, jitter and high risk of dropping, which degrade the service quality. Therefore, handover rate especially vertical handover rate is one of the most metrics to instruct the deployment of mobility management and resource management.

The mathematical expression of handover rate in the homogeneous cellular network has been derived by the regular hexagon coverage model [?] and stochastic geometry model [?] [?]. But for the heterogeneous cellular network, there has not been any reliable and generalized handover expression, due to the randomness of the BS positions of HCNs and the different transmitting power of different tiers. In the real deployment, the HCN BSs are distributed irregularly, sometimes in an anywhere plug-and-play manner, which results in a high level of spatial position randomness. BSs in different tiers have different transmitting power for communication, leading to different cell size for different tiers. As a consequence, it is difficult to characterize the cell boundaries and to track boundary crossings by UEs (i.e. handovers) in the global networks. Few previous works have resolved the above challenges and give a reliable handover rate expression.

In the recent years, stochastic geometry modeling has shown its admirable ability of modeling the position

distribution of HCN BSs [?] [?], and has provided tractable accurate performance bounds for cellular wireless networks. Stochastic geometry is a very powerful mathematical and statistical tool for the modeling, analysis and design of HCNs with random topologies. For instance, the modeling is employed for the capacity analysis of random channel access schemes like ALOHA [?] and carrier sensing multiple access (CSMA) [?], the capacity analysis of single and multiple tier cellular networks [?] and the capacity analysis of cognitive-based networks [?].

Hence, in this paper, we investigate the handover rates including horizontal and vertical handover rates by the stochastic geometry modeling. By employing an infinitesimal method, the mathematical expression of instantaneous handover rate is derived for a typical moving UE. From the derivation, we find that the instantaneous handover rate is related to the instantaneous moving speed, and is independent of the moving direction. That means only the moving speed distribution of the memoryless walking model contributes to the handover rates and the handover rates can be derived through averaging the instantaneous handover rates by the moving speed distribution. Thus, the derived handover rate expressions are applicable for all the memoryless walking models. The derived expressions are validated by computer simulation with multiple walking models and the impacts of system parameters like BS density, transmit power, moving velocity of UE, path loss factor are evaluated. Although our analysis focuses on the scenario of multi-tier HCNs, the analytical framework can be easily extended for more complex scenarios.

2 Downlink System Model

A fairly general model of HCNs considered in this paper contains N tiers of BSs that are distinguished by their spatial densities, transmit powers, path loss exponents and biasing factors. For instance, as shown in Figure 1, high-power macrocell BS networks are overlaid with successively denser and lower power picocells and femtocells. Macrocell BSs and femtocell BSs can be well modeled by spatial random processes [?] [?]. Under this model, the positions of BSs in the n th-tier are modeled according to a homogeneous PPP (Poisson point process) Φ_n with intensity λ_n in an Euclidean plane.

Each BS in the n th-tier has the same transmit power P_n , and has the same path loss exponents $\alpha_n > 2$, $n = 1, \dots, N$. Assume that UEs are uniformly distributed in the Euclidean plane with density of f_u , and the movements of UEs are memoryless and are independent of the distributions of BSs. Memoryless here means the current position of a UE is only related to its latest position and is independent of its more earlier position, i.e. $\mathbb{P}[S(t_0)|S(t_1), S(t_2), \dots] = \mathbb{P}[S(t_0)|S(t_1)]$, where $\mathbb{P}[x]$ is the probability of x , $S(t_i)$ is the position of a UE at time t_i , and $t_i > t_{i+1}$, $i = 0, 1, 2, \dots$.

We assume open access which means a user is allowed to access any tier's BSs. And consider a cell association based on maximum biased-received-power (BRP) (termed biased association), where a mobile UE is associated with the strongest BS in terms of long-term averaged BRP at the UE. The BRP from the j th BS in the n th-tier is $P_{r,nj}$ that can be given by

$$P_{r,nj} = P_n L_0 (R_{nj}/r_0)^{-\alpha_n} B_n \quad (1)$$

where R_{nj} is the distance of the j th BSs in the n th-tier from the origin, L_0 is the path loss at the reference distance r_0 (typically about $(4\pi/\nu)^{-2}$ for $r_0 = 1$, where ν denotes the wavelength). And B_n is the bias factor of admission [?], that could extends the cell range (or coverage) of the n th-tier by employing $B_n > 1$. The considered BRP is a long-term averaged value and fading is averaged out, and so does not include fading. We assume that handover happens only when the UE is going across the boundary of the current BS's coverage, which is determined by the long-term averaged BRP and is shown as Figure 1.

3 Problem Formulation

Consider a typical UE that is at the origin and is admitted to the k th BS of the m th-tier initially. As the typical UE moves, it may immigrate to other BSs in the same tier or other tiers. So its admission state can be depicted as Figure 2, that at time t , the typical UE is admitted to the k th BS with probability $P_{a,k}(t)$, or is admitted to other BS with probability $P_{a,k}(0) - P_{a,k}(t)$. Thus, the instantaneous transition rate from k state to the \bar{k} state at time t is

$$H_k^m(t) = -\frac{dP_{a,k}(t)}{dt} \quad (2)$$

Since the movement of the typical UE is a memoryless process and is independent of BSs' distributions, then the instantaneous transition rate is stable and can be given by

$$H_k^m \triangleq -\lim_{t \rightarrow 0} \frac{dP_{a,k}(t)}{dt} \quad (3)$$

H_k^m is the instantaneous handover rate of the typical UE indeed. Then, the handover rate in a region with area S can be given by

$$\lambda_h = \mathbb{E}\left[\sum_{m=1}^N H_k^m f_u S\right] = \sum_{m=1}^N (\mathbb{E}[H_k^m]) f_u S \quad (4)$$

where $\mathbb{E}[x]$ is the expectation of variable x , and $\mathbb{E}[H_k^m]$ is the average handover rate of the typical UE. As the handover is assumed to happen at the boundary of BS coverage, not at the boundary of a specified region with area S , the average handover rate is independent of the shape of the specified region. In the following section, we would derive the arithmetic expression of $\mathbb{E}[H_k^m]$.

4 Derivation of Handover Rate

In this section, we would derive $\mathbb{E}[H_k^m]$ in two steps: firstly, we use an infinitesimal method to derive the instantaneous handover rate at time 0 of the typical UE with instantaneous velocity v , then average the instantaneous handover rate by the distribution of the velocity v . Thus, the impacts of walking model on the handover rate can be reflected by the distribution of the velocity v , i.e. the walking model decides the distribution of velocity v , then further affects the average handover rate. So the analysis is applicable for all the memoryless walking models with that thought. Note that the moving direction of the typical UE would not affect the instantaneous handover rate and we would give the explanation in the following derivation.

In the following, we would give the position distribution of the associated BS firstly, and then derive the handover probability of $(P_{a,k}(0) - P_{a,k}(t))$, the instantaneous handover rate (H_k^m) and the average handover rate $(\mathbb{E}[H_k^m])$ in turn.

4.1 Position Distribution of Associated BS

Denote (R_{nj}, θ_{nj}) as the polar coordinate of the j th BS in the n th-tier. Assume that the typical UE is admitted to the k th BS of the m th-tier initially, thus $P_{r,mk} > P_{r,nj}$ for all $n \in \{1, \dots, N\}$. According to the max-BRP based association and the BRP definition in equation (1), the distance boundary condition of these unassociated BSs can be derived as equation (5) based on $P_{r,mk} > P_{r,nj}$,

$$R_{nj} > \left(\frac{P_n B_n}{P_m B_m}\right)^{\frac{1}{\alpha_n}} (R_{mk})^{\frac{\alpha_m}{\alpha_n}} \triangleq R_n^{lb} \quad (5)$$

where R_n^{lb} is defined as the distance lower bound of the n th-tier BSs for the clarity of expression.

According to the distance lower bound of each tier, the probability density function (PDF) of the associated BS's distance (R_{mk}) can be derived by using the null probability of a 2-D Poisson process with density λ in an area A , which is $\exp(-\lambda A)$. By setting $\lambda = \lambda_n$ and $A = \pi(R_n^{lb})^2$ for the n th tier, ($n=1, \dots, N$) [?], we could give the probability density function (PDF) of R_{mk} as

$$f(R_{mk}) = 2\pi\lambda_m R_{mk} \exp\left\{-\pi \sum_{n=1}^N \lambda_n (R_n^{lb})^2\right\} \quad (6)$$

where $R_m^{lb} = R_{mk}$. Note that $\int_0^\infty f(R_{mk}) dR_{mk} \triangleq \gamma_m$ is the probability that the typical UE is admitted to a m th-tier BS.

Since BSs are deployed as a PPP (Poisson point process), the θ_{mk} is uniformly distributed in the range of $[0, 2\pi]$, and its PDF is given by

$$f(\theta_{mk}) = \frac{1}{2\pi} \quad (7)$$

Since the distributions of θ_{mk} and R_{mk} are independent of the coordinate axis and the moving direction of UE, we can assume that the X axis is along the moving direction of the typical UE at time 0.

4.2 Derivation of Handover Probability

When the typical UE moves an infinitesimal distance of r , i.e. the typical UE moves to the point $(r, 0)$ with $r \rightarrow 0$, the BRP from the j th BS in the n th-tier is

$$P_{r,nj}^{new} = \frac{P_n B_n L_0}{(\sqrt{(R_{nj} \cos(\theta_{nj}) - r)^2 + (R_{nj} \sin(\theta_{nj}))^2})^{\alpha_n}} \quad (8)$$

According to the max-BRP based association, after the typical UE moves to the new point, it is still admitted to the primary BS k only when the BRP from BS k is larger than the BRP from anyone else. Given the position of BS k , the probability that the typical UE keeps the primary link to the BS k is denoted by $P_{a,k}(r|R_{mk}, \theta_{mk})$ and is given by equation (9).

$$P_{a,k}(r|R_{mk}, \theta_{mk}) = \prod_{n=1}^N \mathbb{P}[P_{r,mk}^{new} \geq P_{r,nj}^{new}] = \prod_{n=1}^N \mathbb{P}[\cos(\theta_{nj}) \leq \frac{R_{nj}^2 + x_{nj}}{2rR_j}] \quad (9)$$

where $\mathbb{P}[x]$ denotes the probability of event x , and x_{nj} is defined as

$$x_{nj} = r^2 - \left(\frac{B_n P_n}{B_m P_m}\right)^{\frac{2}{\alpha_n}} (R_{mk}^2 - 2rR_{mk} \cos(\theta_{mk}) + r^2)^{\frac{\alpha_m}{\alpha_n}} \quad (10)$$

And $\cos(\theta_{nj}) \leq \frac{R_{nj}^2 + x_{nj}}{2rR_j}$ is derived according to $P_{r,mk}^{new} \geq P_{r,nj}^{new}$ in equation (9).

Hence, according to equation (9), the typical UE would keep its primary link if all the BSs in the n th-tier ($n=1, \dots, N$) are in the region of $\cos(\theta_{nj}) \leq \frac{R_{nj}^2 + x_{nj}}{2rR_j}$, or would not otherwise.

We call the region of $\cos(\theta_{nj}) > \frac{R_{nj}^2 + x_{nj}}{2rR_j}$ as the bad region of n th-tier BSs, which is shown as Figure 3. It means that if there are n th-tier BSs in the n th-tier bad region, the typical UE would immigrate from the serving BS to one of those BSs. Thus, $P_{a,k}(r|R_k, \theta_k)$ equals to the probability that no BS is in its bad region for all tiers.

Denote the area of the n th-tier bad region as $A_{mn}(r, R_{mk}, \theta_{mk})$, then we can give the null probability of the PPP Φ_n in the n th-tier bad region as $\exp(-\lambda_n A_{mn}(r, R_{mk}, \theta_{mk}))$. Since all the PPPs $\{\Phi_n\}_{n=1, \dots, N}$ are independent, $P_{a,k}(r|R_k, \theta_k)$ is the product of those null probabilities and can be given by the following equation

$$P_{a,k}(r|R_k, \theta_k) = \prod_{n=1}^N \exp(-\lambda_n A_{mn}(r, R_{mk}, \theta_{mk})) \quad (11)$$

According to $\cos(\cdot) \leq 1$ and the definition of bad region, the θ_{nj} boundary conditions of the n th-tier bad region can be given as

$$1 \geq \cos(\theta_{nj}) > \frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}, \quad (12)$$

Thus, based on the R_{nj} boundary condition in equation (5) and θ_{nj} boundary conditions in equation (12), we can further derive the boundary conditions of the n th-tier bad region in the Appendix 7.1, and give the results as equation (13), where ϑ_{nj} is defined as $\vartheta_{nj} = \theta_{nj}^{max} - \theta_{nj}^{min}$, θ_{nj}^{max} and θ_{nj}^{min} are the upper bound and lower bound of θ_{nj} , respectively. The shapes of bad regions with different boundary conditions in equation (13) can be depicted as Figure 3.

As Figure 3 (a) shows, when $R_n^{lb} < R_{mk}$ holds, according to the derivation in Appendix 7.1, the range of R_{nj} is $[R_n^{lb}, r + \sqrt{r^2 - x_{nj}}]$ and the range of θ_{nj} is $[-\arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}), \arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}})]$ for a certain R_{nj} .

As Figure 3 (b) shows, when both $R_n^{lb} > R_{mk}$ and $|\theta_{mk}| < \arccos(\frac{-R_{mk}}{R_n^{lb}})$ hold, the ranges of R_{nj} and θ_{nj} are the same as the ranges of R_{nj} and θ_{nj} in Figure 3 (a). When both $R_n^{lb} > R_{mk}$ and $|\theta_{mk}| > \arccos(\frac{-R_{mk}}{R_n^{lb}})$ hold, as shown by Figure 3 (c), according to the analysis in Appendix 7.1, the range of R_{nj} is $[R_n^{lb}, r + \sqrt{r^2 - x_{nj}}]$, and the range of θ_{nj} is $[-\pi, \pi]$ if $R_{nj} \in [R_n^{lb}, -r + \sqrt{r^2 - x_{nj}}]$, or is $[-\arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}), \arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}})]$ if $R_{nj} \in [-r + \sqrt{r^2 - x_{nj}}, r + \sqrt{r^2 - x_{nj}}]$.

$$\begin{cases} R_{nj} \in \emptyset, \vartheta_{nj} = 0, & \cos(\theta_{mk}) > \frac{R_{mk}}{R_n^{lb}} \\ R_{nj} \in [R_n^{lb}, r + \sqrt{r^2 - x_{nj}}], \vartheta_{nj} = \begin{cases} 2\pi, & R_{nj} \in [R_n^{lb}, -r + \sqrt{r^2 - x_{nj}}] \\ 2 \arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}), & R_{nj} \in [-r + \sqrt{r^2 - x_{nj}}, r + \sqrt{r^2 - x_{nj}}] \end{cases} & \cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}} \\ R_{nj} \in (R_n^{lb}, r + \sqrt{r^2 - x_{nj}}), \vartheta_{nj} = 2 \arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}), & -\frac{R_{mk}}{R_n^{lb}} \leq \cos(\theta_{mk}) \leq \frac{R_{mk}}{R_n^{lb}} \end{cases} \quad (13)$$

Based on the boundary conditions, the area of the n th-tier bad region ($A_{mn}(r, R_{mk}, \theta_{mk})$) can be derived as the equation (14), where $I(c)$ is the index function that equals 1 if the condition c holds or 0 otherwise. The first term in equation (14) corresponds to the case of $\frac{-R_{mk}}{R_n^{lb}} < \cos(\theta_{mk}) < \frac{R_{mk}}{R_n^{lb}}$, which is depicted by Figure 3 (a) and (b), and the second and third terms correspond to the case of $\cos(\theta_{mk}) < \frac{-R_{mk}}{R_n^{lb}}$ that is depicted by Figure 3 (c).

$$\begin{aligned} A_{mn}(r, R_{mk}, \theta_{mk}) = & \left[\int_{R_n^{lb}}^{r + \sqrt{r^2 - x_{nj}}} 2 \arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}) R_{nj} dR_{nj} \right] I(-\frac{R_{mk}}{R_n^{lb}} \leq \cos(\theta_{mk}) \leq \frac{R_{mk}}{R_n^{lb}}) \\ & + \left[\int_{-r + \sqrt{r^2 - x_{nj}}}^{r + \sqrt{r^2 - x_{nj}}} 2 \arccos(\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}) R_{nj} dR_{nj} + \int_{R_n^{lb}}^{-r + \sqrt{r^2 - x_{nj}}} 2\pi R_{nj} dR_{nj} \right] I(\cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}}) \end{aligned} \quad (14)$$

Thus, the handover probability can be obtained as $(P_{a,k}(0) - \mathbb{E}_{\{R_{mk}, \theta_{mk}\}} P_{a,k}(r | R_{mk}, \theta_{mk}))$.

4.3 Handover Rate Derivation

Since the instantaneous handover rate H_k^m is the derivative of handover probability, according to equations (3) and (11), the instantaneous handover rate H_k^m can be derived as equation (15).

$$\begin{aligned}
H_k^m &= -\lim_{t \rightarrow 0} \frac{dP_{a,k}(t)}{dt} = -\lim_{t \rightarrow 0} \frac{d\mathbb{E}_{\{R_{mk}, \theta_{mk}\}}[P_{a,k}(r|R_{mk}, \theta_{mk})]}{dt} \\
&= -\mathbb{E}_{\{R_{mk}, \theta_{mk}\}} \left[\lim_{t \rightarrow 0} \frac{dP_{a,k}(r|R_{mk}, \theta_{mk})}{dt} \right] \\
&= -\mathbb{E}_{\{R_{mk}, \theta_{mk}\}} \left[\lim_{r \rightarrow 0} \left(\frac{dP_{a,k}(r|R_{mk}, \theta_{mk})}{dr} \frac{dr}{dt} \right) \right] \\
&\stackrel{(a)}{=} -\mathbb{E}_{\{R_{mk}, \theta_{mk}\}} \left[\lim_{r \rightarrow 0} \frac{d}{dr} \left(-\sum_{n=1}^N \lambda_n A_{mn}(r, R_{mk}, \theta_{mk}) \cdot \exp\left(-\sum_{n=1}^N \lambda_n A_{mn}(r, R_{mk}, \theta_{mk})\right) \right) \right] \cdot v \\
&\stackrel{(b)}{=} \mathbb{E}_{\{R_{mk}, \theta_{mk}\}} \left[\sum_{n=1}^N \left(\lambda_n \cdot \lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr} \right) \right] \cdot v
\end{aligned} \tag{15}$$

where $v = \lim_{r \rightarrow 0} \frac{dr}{dt}$ is the instantaneous velocity of the typical UE at time $t = 0$, (a) is obtained according to the expression of $P_{a,k}(r|R_{mk}, \theta_{mk})$ in equation (11), and (b) is obtained due to $\lim_{r \rightarrow 0} A_{mn}(r, R_{mk}, \theta_{mk}) = 0$ and $\lim_{r \rightarrow 0} \exp\left(-\sum_{n=1}^N \lambda_n A_{mn}(r, R_{mk}, \theta_{mk})\right) = 1$.

Denote H_k^{m-n} as the instantaneous handover rate from the k th BS in the m th-tier to the BSs in the n th-tier. Thus

$$H_k^m = \sum_{n=1}^N H_k^{m-n} \tag{16}$$

According to the derivation of H_k^m and the expression of H_k^m in equation (15), we can give H_k^{m-n} as

$$H_k^{m-n} = \mathbb{E}_{\{R_{mk}, \theta_{mk}\}} \left[\lambda_n \cdot \lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr} \right] \cdot v \tag{17}$$

Based on the expression of $A_{mn}(r, R_{mk}, \theta_{mk})$ in equation (14), $\lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr}$ is derived in Appendix 7.2, and the result is given as equation (18).

$$\begin{aligned}
\lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr} &= I(\cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}}) \left[-2\pi \frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk}) \right] \\
&+ I\left(-\frac{R_{mk}}{R_n^{lb}} < \cos(\theta_{mk}) < \frac{R_{mk}}{R_n^{lb}}\right) \left[-2\frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk}) \arccos\left(\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk})\right) + 2\sqrt{(R_n^{lb})^2 - \left(\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk})\right)^2} \right]
\end{aligned} \tag{18}$$

Theorem 1. *The instantaneous handover rate from m th-tier BSs to n th-tier BSs for a UE with instantaneous velocity v is H_k^{m-n} and is given by equation (19).*

$$H_k^{m-n} = 8\lambda_n \lambda_m v \int_0^{+\infty} \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_n^{lb})^2 \exp\left\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\right\} dR_{mk} \tag{19}$$

Proof. Based on equations (17) and (18), H_k^{m-n} is further derived in Appendix 7.3. \square

Hence, the average handover arrival rate λ_h in equation (4) can be given by equation (20), where $f_v(v)$ is the probability density function of the velocity v in the specified region with area S and is determined by the walking model of UEs.

$$\begin{aligned}\lambda_h &= \sum_{m=1}^N (\mathbb{E}[H_k^m f_u S]) = \sum_{m=1}^N \sum_{n=1}^N \mathbb{E}[H_k^{m-n} f_u S] \\ &= f_u S \int_0^{+\infty} v f_v(v) dv \sum_{m=1}^N \sum_{n=1}^N \{8\lambda_n \lambda_m \int_0^{+\infty} \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_n^{lb})^2 \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \} \\ &\quad (20)\end{aligned}$$

Similarly, the average handover arrival rate from m th-tier BSs to n th-tier BSs (denoted as λ_h^{m-n}) can be derived as equation (21).

$$\begin{aligned}\lambda_h^{m-n} &= \mathbb{E}[H_k^{m-n} f_u S] \\ &= f_u S \int_0^{+\infty} v f_v(v) dv 8\lambda_n \lambda_m \int_0^{+\infty} \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_n^{lb})^2 \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \\ &\quad (21)\end{aligned}$$

Although the derived handover rate expressions are not closed-form, these expressions can be efficiently computed numerically as opposed to the usual Monte Carlo methods that rely on repeated random sampling to compute these results.

4.4 Discussions of Handover Rates

In equations of (20) and (21), the general expressions of handover rates have been derived. In this section, some corollaries and special cases of handover rates in the stochastic modeling of the HCNs would be given.

Corollary 1. $\lambda_h^{m-n} = \lambda_h^{n-m}$ holds for any $m, n \in \{1, \dots, N\}$, that is, the forward and reverse handover rates between any two tiers are the same.

Proof. See Appendix 7.4. \square

Corollary 1 holds under the condition that the UE movements in different tiers are homogeneous and UEs are uniformly distributed. Corollary 1 indicates that the mobility between any two tiers would reach statical balance over time.

Corollary 2. When $N = 1$, all the BSs are homogeneous and $\{\lambda_n\} = \lambda$, the expression of average handover rate λ_h can be further simplified as

$$\lambda_h(N = 1) = \frac{4\sqrt{\lambda}}{\pi} f_u S \int_0^{+\infty} v f_v(v) dv \quad (22)$$

Proof. When $N = 1$, according to equation of (20), λ_h can be further derived as equation (23), where (a) is obtained due to $R_m^{lb} = R_{mk}$.

$$\begin{aligned} \lambda_h(N = 1) &= \sum_{m=1}^N (\mathbb{E}[H_k^m f_u S]) \stackrel{(a)}{=} f_u S \int_0^{+\infty} v f_v(v) dv 8\lambda^2 \int_0^{+\infty} \int_0^1 2dz R_{mk}^2 \exp\{-\pi\lambda R_{mk}^2\} dR_{mk} \\ &= f_u S \int_0^{+\infty} v f_v(v) dv 16\lambda^2 \int_0^{+\infty} R_{mk}^2 \exp\{-\pi\lambda R_{mk}^2\} dR_{mk} \\ &= f_u S \int_0^{+\infty} v f_v(v) dv \frac{8\lambda}{\pi\sqrt{\lambda}} \mathcal{Q}(0) \\ &= \frac{4\sqrt{\lambda}}{\pi} f_u S \int_0^{+\infty} v f_v(v) dv \end{aligned} \quad (23)$$

□

Corollary 2 is consistent with the handover rate expression of homogeneous cellular network given in [?].

Corollary 3. When $\{\alpha_n\} = \alpha$, i.e. BSs in all the tiers have the same path loss exponent, the handover rate of λ_h^{m-n} can be simplified as equation (24),

$$\lambda_h^{m-n}(\{\alpha_n\} = \alpha) = \frac{2\lambda_n \lambda_m \beta_n^2 f_u S}{\pi (\sum_{i=1}^N \lambda_i \beta_i^2)^{1.5}} \int_0^{+\infty} v f_v(v) dv \cdot \int_0^{\min(1, \beta_n)} \left[\sqrt{\frac{1-z^2}{\beta_n^2 - z^2}} + \sqrt{\frac{\beta_n^2 - z^2}{1-z^2}} \right] dz \quad (24)$$

where $\beta_n \triangleq (\frac{P_n B_n}{P_m B_m})^{\frac{1}{\alpha}}$.

Proof. When $\{\alpha_n\} = \alpha$, according to equation (5), R_n^{lb} can be simplified as $R_n^{lb} = (\frac{P_n B_n}{P_m B_m})^{\frac{1}{\alpha}} R_{mk} = \beta_n R_{mk}$.

Then λ_h^{m-n} in equation (21) can be further derived as equation (25).

$$\begin{aligned} \lambda_h^{m-n}(\{\alpha_n\} = \alpha) &= f_u S \int_0^{+\infty} v f_v(v) dv 8\lambda_n \lambda_m \int_0^{+\infty} \int_0^{\min(1, \beta_n)} \left[\sqrt{\frac{1-z^2}{\beta_n^2 - z^2}} + \sqrt{\frac{\beta_n^2 - z^2}{1-z^2}} \right] dz \beta_n^2 R_{mk}^2 \exp\{-\pi(\sum_{i=1}^N \lambda_i \beta_i^2) R_{mk}^2\} dR_{mk} \\ &= f_u S \int_0^{+\infty} v f_v(v) dv 8\lambda_n \lambda_m \beta_n^2 \int_0^{\min(1, \beta_n)} \left[\sqrt{\frac{1-z^2}{\beta_n^2 - z^2}} + \sqrt{\frac{\beta_n^2 - z^2}{1-z^2}} \right] dz \int_0^{+\infty} R_{mk}^2 \exp\{-\pi(\sum_{i=1}^N \lambda_i \beta_i^2) R_{mk}^2\} dR_{mk} \\ &= f_u S \int_0^{+\infty} v f_v(v) dv 8\lambda_n \lambda_m \beta_n^2 \int_0^{\min(1, \beta_n)} \left[\sqrt{\frac{1-z^2}{\beta_n^2 - z^2}} + \sqrt{\frac{\beta_n^2 - z^2}{1-z^2}} \right] dz \frac{1}{4\pi (\sum_{i=1}^N \lambda_i \beta_i^2)^{1.5}} \\ &= \frac{2\lambda_n \lambda_m \beta_n^2 f_u S}{\pi (\sum_{i=1}^N \lambda_i \beta_i^2)^{1.5}} \int_0^{+\infty} v f_v(v) dv \int_0^{\min(1, \beta_n)} \left[\sqrt{\frac{1-z^2}{\beta_n^2 - z^2}} + \sqrt{\frac{\beta_n^2 - z^2}{1-z^2}} \right] dz \end{aligned} \quad (25)$$

□

Corollary 4. For a UE with constant velocity v , its residence time in a BS coverage of m th-tier (denoted by T_r^m) is exponential distributed with average value $\frac{\gamma_m}{H_k^m}$, i.e. the PDF of T_r^m can be given by

$$f(T_r^m) = \frac{H_k^m}{\gamma_m} \exp(-\frac{H_k^m}{\gamma_m} T_r^m) \quad (26)$$

where γ_m is the probability that a UE is associated with the m th-tier BS and is given by equation (27) referred to [?].

$$\gamma_m = 2\pi\lambda_m \int_0^\infty R_{mk} \exp\{-\pi \sum_{n=1}^N \lambda_n (R_n^{lb})^2\} dR_{mk} \quad (27)$$

Proof. When the velocity v is constant, the instantaneous handover rate H_k^m would keep constant and do not change with time. So the transition rate from k state to \bar{k} in Figure 2 is constant, i.e. $\frac{1}{\gamma_m} \lim_{t \rightarrow t_0} \frac{dP_{a,k}(t-t_0)}{dt} = -\frac{H_k^m}{\gamma_m}$ is constant for all $t_0 \geq 0$. Thus

$$P_{a,k}(t) = \gamma_m \exp(-\frac{H_k^m}{\gamma_m} t) \quad (28)$$

due to $P_{a,k}(0) = \gamma_m$. So,

$$f(T_r^m) = -\frac{1}{\gamma_m} \frac{P_{a,k}(t)}{dt} = \frac{H_k^m}{\gamma_m} \exp(-\frac{H_k^m}{\gamma_m} T_r^m) \quad (29)$$

□

Corollary 5. When the UEs are not uniformly distributed in the whole region and UEs in each tier BS coverage are uniformly distributed, the handover rate from m th-tier BS to n th-tier BS in a specified region with area S can be given by equation (30).

$$\begin{aligned} \lambda_h^{m-n}(\{f_{m,u}\} \neq f_u) &= \mathbb{E}[H_k^{m-n} f_{m,u} S] \\ &= f_{m,u} S \int_0^{+\infty} v f_v(v) dv 8\lambda_n \lambda_m \int_0^{+\infty} \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_n^{lb})^2 \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \end{aligned} \quad (30)$$

where $f_{m,u}$ is the UE density in the m th tier BS coverage. And the total handover rate in the specified region is $\lambda_h(\{f_{m,u}\} \neq f_u) = \sum_{m=1}^N \sum_{n=1}^N \lambda_h^{m-n}(\{f_{m,u}\} \neq f_u)$.

Proof. For a UE admitted to a m th-tier BS, its average handover rate to a n th-tier BS is $\frac{H_k^m}{\gamma_m}$. On the other hand, the average coverage area of m th-tier BSs in the specified region is $S\gamma_m$. So the total handover rate from m th-tier BSs to n th-tier BSs in the specified region is $\lambda_h^{m-n} = \mathbb{E}[\frac{H_k^{m-n}}{\gamma_m} f_{m,u} S\gamma_m] = \mathbb{E}[H_k^{m-n} f_{m,u} S]$. □

Corollary 6. *When the walking models of UEs in different tier BS coverage are different, the handover rate from the m th-tier BS to n th-tier BS in a specified region with area S can be given by equation (31).*

$$\begin{aligned} \lambda_h^{m-n}(\{f_{m,v}(v)\} \neq f_v(v)) &= \mathbb{E}[H_k^{m-n} f_u S] \\ &= f_u S \int_0^{+\infty} v f_{m,v}(v) dv 8 \lambda_n \lambda_m \int_0^{+\infty} \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_n^{lb})^2 \exp\left\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\right\} dR_{mk} \end{aligned} \quad (31)$$

where $f_{m,v}(v)$ is the velocity distribution in the m th-tier BS coverage. And the total handover rate in the specified region is $\lambda_h(\{f_{m,v}(v)\} \neq f_v(v)) = \sum_{m=1}^N \sum_{n=1}^N \lambda_h^{m-n}(\{f_{m,v}(v)\} \neq f_v(v))$.

Proof. According to the derivation of H_k^{m-n} , H_k^{m-n} is only related to the velocity distribution in the m th-tier BS coverage and is independent of the velocity distribution in other tier BS coverage. Thus, Corollary 6 is can be directly derived from the derivation of H_k^{m-n} in the former section. \square

From the above analysis, we could give the general steps for handover rate analysis by stochastic geometry modeling: firstly, obtain the PDF of the associated BS's position, secondly, by the infinitesimal method, get the area of the bad region, based on which, then derive the instantaneous handover probability, and derive the instantaneous handover rate through taking the derivative, and at last, average the instantaneous handover rate by the distribution of UE's velocity.

5 Simulation Results

5.1 Validation of the Analysis

Now that we have developed the general expression of handover rate for HCNs by stochastic geometry modeling, it is important to see how well the analytical results match the computer simulation. Here, we consider two types of walking models, i.e. straight-line walking model and RWP walking model. For the straight-line walking model, UE would move without changing its moving direction, while for the RWP model, UE would change its moving direction for a randomly chosen direction of $[0, 2\pi]$ and then keep the direction for a randomly chosen duration of $[0, 100s]$. For both models, the velocity is uniformly distributed in $[0, 2v]$.

In the simulation, we considered a circle region with radius of 10Km, generated BSs of each tier and their positions by the PPP Φ_n with the density of λ_n for each try, and generated the UEs with the density of f_u . For each try, all the UEs move with a specified walking model for 10^4 seconds. We counted the number of handover in a specified region with area $S = 1\text{km}^2$ and gave the handover rates by averaging the results.

A total of two tiers are modeled according to PPP in the simulation ($N=2$). For the simulation, some parameters keep constant and these parameters are $f_u = 100/\text{km}^2$, $S = 1\text{km}^2$, $P_1 = 1$, $P_2 = 0.2$, $B_1 = 1$, $\alpha_1 = 3.5$ and $\lambda_1 = 1/\text{km}^2$. Other parameters would vary in different tries and these parameters are the velocity v , the 2nd-tier BS density λ_2 and, 2nd-tier BS's path loss factor α_2 and the 2nd-tier BS's bias factor B_2 .

Figure 4 compares the analytical and experimental handover rates of λ_h^{1-1} , λ_h^{1-2} and λ_h^{2-2} . In Figure 4, the dotted lines represent the analytical results, the circles and triangles represent the simulation results obtained by straight-line walking model and RWP walking model, respectively. And we set $\lambda_2 = 2/\text{km}^2$, $B_2 = 1$ and $\alpha_2 = 3.5$. In Figure 4, it is observed that the handover rates of λ_h^{1-1} , λ_h^{1-2} and λ_h^{2-2} increase linearly with the average velocity of UE. This matches the derived expression of handover rate. In Figure 4, we can see that λ_h^{1-1} is the largest, λ_h^{1-2} takes the second place and λ_h^{2-2} is the minimum for any value of average velocity. This can be explained as follows. Since $P_1 = 1$ and $P_2 = 0.2$, the coverage of 1st-tier BS is much larger than the coverage of 2nd-tier BS. So the 1st-tier BS has longer boundary line than the 2nd-tier BS. Thus, the total length of the boundary line between two 1st-tier BSs is the longest, the total length of the boundary line between a 1st-tier BS and a 2nd-tier BS is shorter and the total length of boundary line between two 2nd-tier BSs is the shortest. Hence, the handover is most likely to happen at the boundary between two 1st-tier BSs, is less likely to happen at the boundary between a 1st-tier BS and a 2nd-tier BS, and is least likely to happen at the boundary between two 2nd-tier BSs. From the figure, it can be seen that the relative error between the analytical results and the simulation results are less than 3% for both walking models. The relative error is most likely brought by the limited simulation time. The good matching validates that the analysis is reliable with the variation of UE velocity. In the simulation, we find that the straight-line walking model and the RWP walking model without pause time almost have the same handover rates when they have the same velocity distribution.

Figure 5 illustrates the analytical and experimental results of the total handover rate (λ_h) and the handover rate between the two tiers (λ_h^{1-2}) with different 2nd-tier BSs density (λ_2) for different average UE velocity (v). From the figure, we can observe that handover rates increase linearly with the average UE velocity, and both λ_h and λ_h^{1-2} increase with the 2nd-tier BS density. This is because larger BS density leads to smaller cells and UEs are more likely to move out the smaller cells. The analytical results match well with experimental results of straight-line walking model and RWP walking model, with relative error less than 3%. This validates that the analysis is reliable with the variation of both BS density and moving velocity.

Figure 6 demonstrates the analytical and experimental results of handover rates with variations of 2nd-tier BS density (λ_2) and path loss factor (α_2). The analytical results match well with the experimental results for both walking models with relative error less than 3%. This validates that the analysis is reliable with the variations of both BS density and path loss factors. From this figure, we can observe that all the handover rates increase with the 2nd-tier BS density. The handover rate of λ_h^{1-1} increases with 2nd-tier BS path loss factor α_2 , λ_h^{1-2} increases with α_2 firstly, then decreases with α_2 , and λ_h^{2-2} decreases with α_2 . This is because the coverage of the 2nd-tier BS decreases with α_2 and the coverage of the 1st-tier BS increases with α_2 according to equation (1). Thus, the boundary line of the 1st-tier BSs increases with the α_2 and the boundary line of the 2nd-tier BSs decreases with the α_2 . So, the total length of the boundary line between two 1st-tier BSs increases with α_2 , the total length of the boundary line between two 2nd-tier BSs decreases with α_2 , and the total length of the boundary line between a 1st-tier BS and a 2nd-tier BS increases firstly with α_2 , then decreases with α_2 . It is the reason that leads to the increasing of λ_h^{1-1} , the decreasing of λ_h^{2-2} , and the changing of λ_h^{1-2} .

Figure 7 demonstrates the experimental results of the forward and reverse handover rates between the two tiers for the RWP walking model with the variations of 2nd-tier BS density, path loss factor and average velocity. Figure 7 shows that the forward handover rates (λ_h^{1-2}) match well with reverse handover rates (λ_h^{2-1}) between the two tiers with relative error less than 3%. This validates the reliability of Corollary 1.

Figure 8 demonstrates the analytical and experimental results of residual time distributions with constant velocity. The analysis results (denoted by ‘Theo.’ in the figure) is calculated by equation (26), and the experimental results (denoted by ‘Simu.’ in the figure) is obtained by the RWP model with constant velocity $v=5\text{m/s}$. From the figure, we can see that the experimental results match well with the analysis results, which validates the effectiveness of Corollary 4. As the figure illustrated, the residual time in the 1st-tier BS is much larger than the residual time in the 2nd-tier BS due to the fact that the 1st-tier BS coverage is much larger than the 2nd-tier BS coverage. As the figure shown, both the residual time in the 1st-tier BS and in the 2nd-tier BS decrease with the 2nd-tier BS density λ_2 , but the decrement of residual time in the 1st-tier BS is more significant. This can be explained as follows. With the increase of 2nd-tier BS density, the average coverage of each tier BS decreases, so the residual time decreases for each tier. On the other hand, with the increase of 2nd-tier BS density, the average 1st-tier BS coverage decreases more significantly because the coverage of each 1st-tier BS is much larger than the 2nd-tier BS coverage and the increased 2nd-tier BSs would be more likely to occupy the 1st-tier BSs coverage. Thus, the decrement of residual time in the 1st-tier BS is more significant.

5.2 Effect of Bias Factor

Figure 9 shows the numerical results of handover rates with the variation of 2nd-tier BS's bias factor (B_2). From this figure, we can observe that the total handover rate (λ_h) decreases with the 2nd-tier BS's bias factor, λ_h^{1-1} and λ_h^{1-2} decrease with B_2 and λ_h^{2-2} increases with B_2 when $B_2 \in [1, 2]$, which is a reasonable range of B_2 . This can be explained as follows. According to equation (20), when R_{mk} equals R_n^{lb} , λ_h^{m-n} could reach the minimal value, and λ_h^{m-n} decreases with R_{mk} if $R_{mk} < R_n^{lb}$ holds. Thus, when B_2 increases and $R_{1k} < R_2^{lb}$ holds, R_{1k} increases and λ_h^{1-2} decreases. When B_2 increases, the coverage of 1st-tier BS decreases and the coverage of 2nd-tier BS increases according to equation (1), so the boundary line of 1st-tier BS decreases and the boundary line of the 2nd-tier BS increases, which lead to the results that λ_h^{1-1} decreases with B_2 and λ_h^{2-2} increases with B_2 . The results demonstrate that we can decrease the total handover rate by reasonably adjusting the bias factors.

6 Conclusion

In the literature, there has not been any general handover rate derivation for the heterogeneous cellular networks. Thus, in this paper, we give a generalized handover analytical framework by employing the stochastic geometry modeling for heterogeneous cellular networks, derive the arithmetic expression of handover rate, give some meaningful corollaries and validate the analysis by computer simulation. The analysis may shed some light for future extension and study.

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7 Appendix

7.1 Boundary conditions of bad region

According to the bad region boundary conditions in equation (12), we can further derive ϑ_{nj} as follow:

$$\begin{cases} \vartheta_{nj} = 0, & \frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} > 1 \\ \vartheta_{nj} = 2 \arccos \frac{R_{nj}^2 + x_{nj}}{2rR_{nj}}, & -1 \leq \frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} \leq 1 \\ \vartheta_{nj} = 2\pi, & \frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} < -1 \end{cases} \quad (32)$$

For the case of $-1 \leq \frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} \leq 1$, we can further derive the inequations of $\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} \leq 1$ and $\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} \geq -1$

as equations (33) and (34), respectively,

$$\begin{aligned} R_{nj}^2 - 2rR_{nj} + x_{nj} &\leq 0 \\ \Rightarrow R_{nj} &\in [r - \sqrt{r^2 - x_{nj}}, r + \sqrt{r^2 - x_{nj}}] \end{aligned} \quad (33)$$

$$\begin{aligned} R_{nj}^2 + 2rR_{nj} + x_{nj} &\geq 0 \\ \Rightarrow R_{nj} &\in (-\infty, -r - \sqrt{r^2 - x_{nj}}) \cup (-r + \sqrt{r^2 - x_{nj}}, +\infty) \end{aligned} \quad (34)$$

Hence, we can give the range of R_{nj} as equation (35) according to equation (5) and $R_{nj} > 0$.

$$\min(R_n^{lb}, |-r + \sqrt{r^2 - x_{nj}}|) \leq R_{nj} \leq r + \sqrt{r^2 - x_{nj}} \quad (35)$$

As $-r + \sqrt{r^2 - x_{nj}} > 0$ when $r \rightarrow 0^+$, we can further simplify equation (35) as

$$\min(R_n^{lb}, -r + \sqrt{r^2 - x_{nj}}) \leq R_{nj} \leq r + \sqrt{r^2 - x_{nj}} \quad (36)$$

For case of $\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} > 1$, similar to equation (33), we can further derive the inequation of $\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} > 1$ as $R_{nj} \in (\max(R_n^{lb}, r + \sqrt{r^2 - x_{nj}}), +\infty)$.

For the case of $\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} < -1$, similar to equation (34), we can further derive the inequation of $\frac{R_{nj}^2 + x_{nj}}{2rR_{nj}} < -1$ as $R_{nj} \in (R_n^{lb}, -r + \sqrt{r^2 - x_{nj}})$

On the other hand, according to the expression of R_n^{lb} in equation (5) and x_{nj} in equation (10), $R_n^{lb} = -r + \sqrt{r^2 - x_{nj}}$ holds when $r = 0$. And the derivative of $-r + \sqrt{r^2 - x_{nj}}$ at $r = 0$ is derived as

$$\lim_{r \rightarrow 0} \frac{d(-r + \sqrt{r^2 - x_{nj}})}{dr} = \left(\frac{B_n P_n}{B_m P_m}\right)^{\frac{1}{\alpha_n}} (R_{mk})^{\frac{\alpha_m}{\alpha_n}} \frac{-\cos(\theta_{mk})}{R_{mk}} - 1 = -\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk}) - 1 \quad (37)$$

Thus, $-r + \sqrt{r^2 - x_{nj}}$ increases at $r = 0$ if $\cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}}$ or decreases otherwise. So,

$$\begin{cases} R_n^{lb} > -r + \sqrt{r^2 - x_{nj}}, & \cos(\theta_{mk}) > -\frac{R_{mk}}{R_n^{lb}} \\ R_n^{lb} < -r + \sqrt{r^2 - x_{nj}}, & \cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}} \end{cases} \quad (38)$$

Similarly, the following relationships can be derived,

$$\begin{cases} R_n^{lb} > r + \sqrt{r^2 - x_{nj}}, & \cos(\theta_{mk}) > \frac{R_{mk}}{R_n^{lb}} \\ R_n^{lb} < r + \sqrt{r^2 - x_{nj}}, & -\frac{R_{mk}}{R_n^{lb}} < \cos(\theta_{mk}) < \frac{R_{mk}}{R_n^{lb}} \end{cases} \quad (39)$$

Based on the above relationships, the boundary conditions of equation (32) can be derived as equation (13).

7.2 Derivation of $\lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr}$

For the clarity of expression, we define the expressions of $A_{mn}^{(1)}(r, R_{mk}, \theta_{mk})$, $A_{mn}^{(2)}(r, R_{mk}, \theta_{mk})$ and $A_{mn}^{(3)}(r, R_{mk}, \theta_{mk})$ in the equation of (40).

$$\begin{aligned}
A_{mn}^{(1)}(r, R_{mk}, \theta_{mk}) &= \left[\int_{R_n^{lb}}^{r+\sqrt{r^2-x_{nj}}} 2 \arccos\left(\frac{R_{nj}^2+x_{nj}}{2rR_{nj}}\right) R_{nj} dR_{nj} \right] I\left(-\frac{R_{mk}}{R_n^{lb}} \leq \cos(\theta_{mk}) \leq \frac{R_{mk}}{R_n^{lb}}\right) \\
A_{mn}^{(2)}(r, R_{mk}, \theta_{mk}) &= \left[\int_{-r+\sqrt{r^2-x_{nj}}}^{r+\sqrt{r^2-x_{nj}}} 2 \arccos\left(\frac{R_{nj}^2+x_{nj}}{2rR_{nj}}\right) R_{nj} dR_{nj} \right] I(\cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}}) \\
A_{mn}^{(3)}(r, R_{mk}, \theta_{mk}) &= \left[\int_{R_n^{lb}}^{-r+\sqrt{r^2-x_{nj}}} 2\pi R_{nj} dR_{nj} \right] I(\cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}})
\end{aligned} \tag{40}$$

Based on those expressions and equation (14), the following relationship can be obtained,

$$A_{mn} = A_{mn}^{(1)} + A_{mn}^{(2)} + A_{mn}^{(3)} \tag{41}$$

Then $\lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr}$ can be derived by deriving $\lim_{r \rightarrow 0} \frac{dA_{mn}^{(i)}(r, R_{mk}, \theta_{mk})}{dr}$, $i = 1, 2, 3$. For simplicity, we define $\varphi^u(r)$, $\varphi^d(r)$ and $f(r, R_{nj})$ as $\varphi^u(r) = r + \sqrt{r^2 - x_{nj}}$, $\varphi^d(r) = -r + \sqrt{r^2 - x_{nj}}$, and $f(r, R_{nj}) = 2R_{nj} \arccos(\frac{x_{nj}+R_{nj}^2}{2rR_{nj}})$, respectively.

$$\begin{aligned}
\lim_{r \rightarrow 0} \frac{dA_{mn}^{(1)}}{dr} &= \lim_{r \rightarrow 0} \frac{d}{dr} \int_{R_n^{lb}}^{\varphi^u(r)} f(r, R_{nj}) dR_{nj} I^{(1)} \\
&= \lim_{r \rightarrow 0} \left[f(r, \varphi^u(r)) \frac{d}{dr} \varphi^u(r) + \int_{R_n^{lb}}^{\varphi^u(r)} \frac{d}{dr} f(r, R_{nj}) dR_{nj} \right] I^{(1)} \\
&\stackrel{(a)}{=} \lim_{r \rightarrow 0} \left[\int_{R_n^{lb}}^{\varphi^u(r)} \frac{d}{dr} f(r, R_{nj}) dR_{nj} \right] I^{(1)} \\
&= \lim_{r \rightarrow 0} \left[\int_{R_n^{lb}}^{\varphi^u(r)} \frac{-2R_{nj}}{\sqrt{1 - (\frac{R_{nj}^2+x_{nj}}{2R_{nj}r})^2}} \frac{\frac{dx_{nj}}{dr} r - (R_{nj}^2+x_{nj})}{2R_{nj}r^2} dR_{nj} \right] I^{(1)} \\
&\stackrel{y=R_{nj}}{=} \lim_{r \rightarrow 0} I^{(1)} \left[\int_{(R_n^{lb})^2}^{(\varphi^u(r))^2} \frac{-1}{\sqrt{1 - \frac{(y+x_{nj})^2}{4y r^2}}} \frac{\frac{dx_{nj}}{dr} r - (y+x_{nj})}{r^2} \frac{1}{2\sqrt{y}} dy \right] \\
&= \lim_{r \rightarrow 0} I^{(1)} \left[\int_{(R_n^{lb})^2}^{(\varphi^u(r))^2} \frac{-1}{\sqrt{4r^4 - 4r^2 x_{nj} - (y+x_{nj}-2r^2)^2}} \cdot \frac{(\frac{dx_{nj}}{dr} r - 2r^2) - (y+x_{nj}-2r^2)}{r} dy \right] \\
&= \lim_{r \rightarrow 0} -I^{(1)} \left[-\frac{\frac{dx_{nj}}{dr} r - 2r^2}{r} \arccos \frac{y+x_{nj}-2r^2}{\sqrt{4r^4 - 4r^2 x_{nj} - (y+x_{nj}-2r^2)^2}} + \frac{1}{r} \sqrt{4r^4 - 4r^2 x_{nj} - (y+x_{nj}-2r^2)^2} \right] \frac{(\varphi^u(r))^2}{(R_n^{lb})^2} \\
&= \lim_{r \rightarrow 0} -I^{(1)} \left[\frac{\frac{dx_{nj}}{dr} r - 2r^2}{r} \arccos\left(\frac{(R_n^{lb})^2+x_{nj}-2r^2}{\sqrt{4r^4 - 4r^2 x_{nj} - ((R_n^{lb})^2+x_{nj}-2r^2)^2}}\right) - \frac{1}{r} \sqrt{4r^4 - 4r^2 x_{nj} - ((R_n^{lb})^2+x_{nj}-2r^2)^2} \right] \\
&\stackrel{(b)}{=} I^{(1)} \left[-\frac{2(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk}) \arccos\left(\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk})\right) + 2\sqrt{(R_n^{lb})^2 - (\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk}))^2} \right]
\end{aligned} \tag{42}$$

where $I^{(1)} = I(-\frac{R_{mk}}{R_n^{lb}} \leq \cos(\theta_{mk}) \leq \frac{R_{mk}}{R_n^{lb}})$, and (a) is given by $f(r, \varphi^u(r)) = 0$, (b) is given due to $\lim_{r \rightarrow 0} x_{nj} = -(R_n^{lb})^2$, $\lim_{r \rightarrow 0} \frac{d}{dr} x_{nj} = \frac{2(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk})$ and the L'Hopital rule.

$$\begin{aligned}
\lim_{r \rightarrow 0} \frac{dA_{mn}^{(2)}}{dr} &= \lim_{r \rightarrow 0} \int_{\varphi^d(r)}^{\varphi^u(r)} f(r, R_{nj}) dR_{nj} I^{(2)} \\
&= \lim_{r \rightarrow 0} [f(r, \varphi^u(r)) \frac{d}{dr} \varphi^u(r) - f(r, \varphi^d(r)) \frac{d}{dr} \varphi^d(r) + \int_{\varphi^d(r)}^{\varphi^u(r)} \frac{d}{dr} f(r, R_{nj}) dR_{nj}] I^{(2)} \\
&\stackrel{(c)}{=} [2\pi R_n^{lb} (\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk}) + 1) - 2\pi \frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk})] I^{(2)}
\end{aligned} \tag{43}$$

where $I^{(2)} = I(\cos(\theta_{mk}) < -\frac{R_{mk}}{R_n^{lb}})$, and (c) is obtained by $f(r, \varphi^u(r)) = 0$, $f(r, \varphi^d(r)) = 2\pi$, $\lim_{r \rightarrow 0} \frac{d}{dr} \varphi^d(r) = -\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk}) - 1$ and $\lim_{r \rightarrow 0} \int_{\varphi^d(r)}^{\varphi^u(r)} \frac{d}{dr} f(r, R_{nj}) dR_{nj} = -2\pi \frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk})$, which can be derived similarly as equation (42).

$$\begin{aligned}
\lim_{r \rightarrow 0} \frac{dA_{mn}^{(3)}}{dr} &= \lim_{r \rightarrow 0} \frac{d}{dr} \int_{R_n^{lb}}^{\varphi^d(r)} 2\pi R_{nj} dR_{nj} I^{(2)} \\
&= \lim_{r \rightarrow 0} [2\pi \varphi^d(r) \frac{d\varphi^d(r)}{dr}] I^{(2)} \\
&= -2\pi R_n^{lb} (\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk}) + 1) I^{(2)}
\end{aligned} \tag{44}$$

which is obtained due to $\lim_{r \rightarrow 0} \varphi^d(r) = R_n^{lb}$.

Thus, add equations of (42) (43) and (44) together, we can give $\lim_{r \rightarrow 0} \frac{dA_{mn}(r, R_{mk}, \theta_{mk})}{dr}$ as equation (18).

7.3 Derivation of H_k^{m-n}

According equation (17), H_k^{m-n} can be further expressed as equation (45),

$$\begin{aligned}
H_k^{m-n} &= \lambda_n v \int_0^{+\infty} \int_0^{2\pi} \lim_{r \rightarrow 0} \frac{dA_{mn}}{dr} f(\theta_{mk}) f(R_{mk}) d\theta_{mk} dR_{mk} \\
&= \lambda_n v \int_0^{+\infty} \int_0^{2\pi} \lim_{r \rightarrow 0} \frac{dA_{mn}}{dr} \lambda_m R_{mk} \exp(-\pi \lambda_m R_{mk}^2) d\theta_{mk} dR_{mk}
\end{aligned} \tag{45}$$

For simplicity, we define $h_{k,1}^{m-n}$, $h_{k,2}^{m-n}$ and $h_{k,3}^{m-n}$ in equation of (46).

$$\begin{aligned}
h_{k,1}^{m-n} &= I^{(2)} [-2\pi \frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk})] \\
h_{k,2}^{m-n} &= I^{(1)} [-2 \frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk}) \arccos(\frac{R_n^{lb}}{R_{mk}} \cos(\theta_{mk}))] \\
h_{k,3}^{m-n} &= I^{(1)} [2 \sqrt{(R_n^{lb})^2 - (\frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk}))^2}]
\end{aligned} \tag{46}$$

Thus

$$H_k^{m-n} = \mathbb{E}[h_{k,1}^{m-n} + h_{k,2}^{m-n} + h_{k,3}^{m-n}] \tag{47}$$

And $\mathbb{E}[h_{k,1}^{m-n}]$, $\mathbb{E}[h_{k,2}^{m-n}]$ and $\mathbb{E}[h_{k,3}^{m-n}]$ can be derived as equations of (48), (49) and (50), respectively.

$$\begin{aligned}
\mathbb{E}[h_{k,1}^{m-n}] &= 2\lambda_n \lambda_m v \int_0^{+\infty} \int_{\pi - \arccos(\min(1, \frac{R_{mk}}{R_n^{lb}}))}^{\pi} -2\pi \frac{(R_n^{lb})^2}{R_{mk}} \cos(\theta_{mk}) R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} d\theta_{mk} dR_{mk} \\
&= 2\lambda_n \lambda_m v \int_0^{+\infty} [2\pi \frac{(R_n^{lb})^2}{R_{mk}} (-\sin(\theta_{mk}))|_{\pi - \arccos(\min(1, \frac{R_{mk}}{R_n^{lb}}))}^{\pi}] R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \\
&= 2\lambda_n \lambda_m v \int_0^{+\infty} [2\pi \frac{(R_n^{lb})^2}{R_{mk}} \sqrt{1 - (\min(1, \frac{R_{mk}}{R_n^{lb}}))^2}] R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk}
\end{aligned} \tag{48}$$

$$\begin{aligned}
\mathbb{E}[h_{k,2}^{m-n}] &\stackrel{z=\cos(\theta_{mk})}{=} 2\lambda_n \lambda_m v \int_0^{+\infty} \int_{\min(1, \frac{R_{mk}}{R_n^{lb}})}^{-\min(1, \frac{R_{mk}}{R_n^{lb}})} [-2\frac{(R_n^{lb})^2}{R_{mk}} z \arccos(\frac{R_n^{lb}}{R_{mk}} z)] \frac{-1}{\sqrt{1-z^2}} dz R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \\
&= 2\lambda_n \lambda_m v \int_0^{+\infty} 2\frac{(R_n^{lb})^2}{R_{mk}} [(-\sqrt{1-z^2} \arccos(\frac{R_n^{lb}}{R_{mk}} z))|_{\min(1, \frac{R_{mk}}{R_n^{lb}})}^{-\min(1, \frac{R_{mk}}{R_n^{lb}})} + 2 \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \frac{\sqrt{1-z^2}}{\sqrt{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}}] R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \\
&= 2\lambda_n \lambda_m v \int_0^{+\infty} 2\frac{(R_n^{lb})^2}{R_{mk}} [-\pi \sqrt{1 - (\min(1, \frac{R_{mk}}{R_n^{lb}}))^2} + 2 \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \frac{\sqrt{1-z^2}}{\sqrt{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}}] R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk}
\end{aligned} \tag{49}$$

$$\begin{aligned}
\mathbb{E}[h_{k,3}^{m-n}] &\stackrel{z=\cos(\theta_{mk})}{=} 2\lambda_n \lambda_m v \int_0^{+\infty} \int_{\min(1, \frac{R_{mk}}{R_n^{lb}})}^{-\min(1, \frac{R_{mk}}{R_n^{lb}})} [2\sqrt{1 - (\frac{R_n^{lb}}{R_{mk}} z)^2} \frac{-1}{\sqrt{1-z^2}} R_n^{lb}] R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \\
&= 2\lambda_n \lambda_m v \int_0^{+\infty} 2\frac{(R_n^{lb})^2}{R_{mk}} [2 \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \frac{\sqrt{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}}{\sqrt{1-z^2}}] R_{mk} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk}
\end{aligned} \tag{50}$$

Thus, H_k^{m-n} can be derived as equation of (19) by adding the results of equations of (48), (49) and (50) together.

7.4 Proof of Corollary 1

According to the equation (21), $\lambda_h^{m-n} = \lambda_h^{n-m}$ can be proved by proving that the equation of $H_k^{m-n} = H_k^{n-m}$ holds. Based on the expression of H_k^{m-n} in equation (19), the proof can be given by the equation (51), where (d) follows from plugging $R_{mk} = x^{\frac{\alpha_n}{\alpha_m}} (\frac{P_n B_m}{P_n B_n})^{\frac{1}{\alpha_m}}$, (e) follows from plugging $z_1 = \frac{R_{m,n}^{lb}}{x} z$, and $R_{i,n}^{lb} = (\frac{P_i B_i}{P_n B_n})^{\frac{1}{\alpha_i}} x^{\frac{\alpha_n}{\alpha_i}}$, (f) follows from plugging $x = R_{nj}$, where R_{nj} is the nearest distance of the n th-tier BSs to the origin.

$$\begin{aligned}
H_k^{m-n} &= 8\lambda_n\lambda_mv \int_0^{+\infty} \int_0^{\min(1, \frac{R_{mk}}{R_n^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{R_{mk}}{R_n^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_n^{lb})^2 \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_i^{lb})^2)\} dR_{mk} \\
&\stackrel{(d)}{=} 8\lambda_n\lambda_mv \int_0^{+\infty} \int_0^{\min(1, \frac{R_{m,n}^{lb}}{x})} \left[\sqrt{\frac{1-z^2}{(\frac{R_{m,n}^{lb}}{x})^2 - z^2}} + \sqrt{\frac{(\frac{R_{m,n}^{lb}}{x})^2 - z^2}{1-z^2}} \right] dz x^{\frac{\alpha_n}{\alpha_m}+1} (\frac{P_m B_m}{P_n B_n})^{\frac{1}{\alpha_m}} \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_{i,n}^{lb})^2)\} dx \\
&\stackrel{(e)}{=} 8\lambda_n\lambda_mv \int_0^{+\infty} \int_0^{\min(1, \frac{x}{R_{m,n}^{lb}})} \left[\sqrt{\frac{(\frac{x}{R_{m,n}^{lb}})^2 - z_1^2}{1-z_1^2}} + \sqrt{\frac{1-z_1^2}{(\frac{x}{R_{m,n}^{lb}})^2 - z_1^2}} \right] dz_1 (R_{m,n}^{lb})^2 \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_{i,n}^{lb})^2)\} dx \\
&\stackrel{z=z_1}{=} 8\lambda_n\lambda_mv \int_0^{+\infty} \int_0^{\min(1, \frac{x}{R_{m,n}^{lb}})} \left[\sqrt{\frac{1-z^2}{(\frac{x}{R_{m,n}^{lb}})^2 - z^2}} + \sqrt{\frac{(\frac{x}{R_{m,n}^{lb}})^2 - z^2}{1-z^2}} \right] dz (R_{m,n}^{lb})^2 \exp\{-\pi \sum_{i=1}^N (\lambda_i (R_{i,n}^{lb})^2)\} dx \\
&\stackrel{(f)}{=} H_k^{n-m}
\end{aligned} \tag{51}$$

Figures

Figure 1 - Example of downlink HCNs with three tiers of BSs: high-power macrocell BSs (red square) are overlaid with successively denser and lower power picocells (red triangle) and femtocells (red circle).

Figure 2 - The admission state and its transition of the typical UE.

Figure 3 - The bad region when the typical UE moves to $(r,0)$.

Figure 4 - The average handover rates between tiers in $1km^2$ region with different \bar{v} .

Figure 5 - The total handover rate (λ_h) and handover rate between different tiers (λ_h^{1-2}) in $1km^2$ region with different average velocity \bar{v} and 2nd-tier BSs density λ_2 .

Figure 6 - The handover rates between tiers in $1km^2$ region with different average velocity \bar{v} and 2nd-tier path loss factor α_2 .

Figure 7 - The forward and reverse handover rates between the two tiers in $1km^2$ region with different average velocity \bar{v} , 2nd-tier BSs density λ_2 and 2nd-tier path loss factor α_2 .

Figure 8 -The CDF of residual time in the 2-tier BSs with different λ_2 .

Figure 9 -The average handover arrival rates in $1km^2$ region with different 2nd-tier bias factor (B_2).